Y12 SPRING 1 – Chp. 7/8/11

The Factor Theorem

- A **polynomial** is a finite expression with whole number indices (e.g. $x^3 - 1$, $x^2 + x - 7x^{-2}$)
- The **Factor Theorem** states that if $f(x)$ is a polynomial, then:

$$\boxed{f(p) = 0 \iff (x-p) \text{ is a factor of } f(x)}$$

- How to use the factor theorem: There are 2 types of question:

① "Show p is a factor of $f(x)$" ← plug p into the polynomial to find $f(p)$, and show that this is $=0$. Then conclude using the factor theorem that $f(p) = 0 \Rightarrow p$ is a factor of $f(x)$.

② "Factorise $f(x)$" ← Guess a factor of $f(x)$ – usually $0, 1, -1, 2$ are good guesses. Then perform polynomial division, and factorise the remaining (usually) quadratic in the normal ways

Proof!

- A **conjecture** is a mathematical statement which is yet to be proven.
- A **theorem** is a mathematical statement which has been proven.

* A proof must ① show all assumptions you are using, have a ② clear sequential list of steps that you logically follow, and must ③ cover all possible cases. You should usually make a ④ concluding statement, e.g. restating the original conjecture that you have now proven.

Methods of Proof: Method 1: Proof by deduction ← generalise mathematical objects (e.g. odd number is $2n+1$) and get from LHS → RHS or RHS → LHS.

Method 2: Proof by Exhaustion ← break down the statement into smaller cases, and prove each one by deduction (or simple demonstration)

Method 3: Dis-proof by Counter-Example ← find a single ~~ex~~ counter-example to disprove the entire statement.

The Binomial Expansion

e.g. $(3x+2)^3 = \binom{3}{0}(3x)^3(2)^0 + \binom{3}{1}(3x)^2(2)^1 + \binom{3}{2}(3x)^1(2)^2 + \binom{3}{3}(3x)^0(2)^3$

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{r}a^{n-r}b^r + \dots + \binom{n}{n}a^0 b^n$$

Notation!

$n! = n \times (n-1) \times \dots \times 3 \times 2 \times 1$ e.g. $4! = 4 \times 3 \times 2 \times 1 = 24$ / $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$

$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$ = "the number of ways of choosing r items from a group of n" e.g. $\binom{6}{2} = {}^6C_2 = \frac{6!}{2!4!} = \frac{720}{2 \times 24} = 15$

Y12 Spring 1 – Chapter 11 – Vectors

A Whereas a **coordinate** represents a **position** in space, a **vector** represents a **displacement** in space.

A vector has 2 properties:
- Direction
- Magnitude (i.e. length)

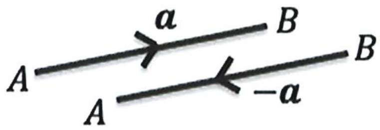
If P and Q are points then $\overrightarrow{PQ}$ is the vector between them.

If $\boldsymbol{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ $|\boldsymbol{a}| = \sqrt{x^2 + y^2}$

B If two vectors $\overrightarrow{PQ}$ and $\overrightarrow{RS}$ have the same magnitude and direction, **they're the same vector** and are **parallel.**

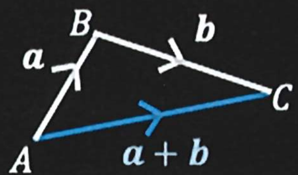
This might seem obvious, but students sometimes think the vector is different because the movement occurred at a different point in space. Nope!

C $\overrightarrow{AB} = -\overrightarrow{BA}$ and the two vectors are parallel, equal in magnitude but in **opposite directions.**



D Triangle Law for vector addition:

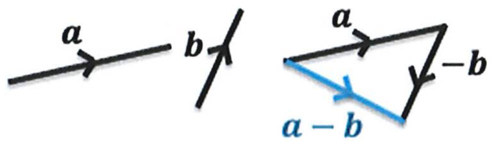
$$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$$



The vector of multiple vectors is known as the **resultant vector.**
(you will encounter this term in Mechanics)

E Vector **subtraction** is defined using vector addition and negation:

$$\boldsymbol{a} - \boldsymbol{b} = \boldsymbol{a} + (-\boldsymbol{b})$$

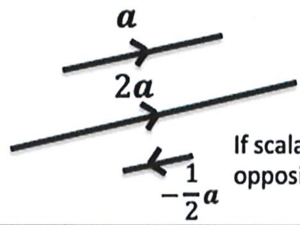


F The zero vector **0** (a bold 0), represents no movement.

$$\overrightarrow{PQ} + \overrightarrow{QP} = \boldsymbol{0}$$

In 2D: $\boldsymbol{0} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

G A **scalar** is a normal number, which can be used to 'scale' a vector.
- The **direction** will be the **same.**
- But the **magnitude** will be **different** (unless the scalar is 1).



If scalar is negative, opposite direction.

H Any vector parallel to the vector $\boldsymbol{a}$ can be written as $\lambda\boldsymbol{a}$, where λ is a scalar.

The implication is that if we can write one vector **as a multiple of** another, then we can show they are parallel.

"Show $2\boldsymbol{a} + 4\boldsymbol{b}$ and $3\boldsymbol{a} + 6\boldsymbol{b}$ are parallel".
$3\boldsymbol{a} + 6\boldsymbol{b} = \frac{3}{2}(\boldsymbol{a} + 2\boldsymbol{b})$ ∴ parallel

I ✎ A **unit vector** is a vector of magnitude 1. $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit vectors in the x-axis and y-axis respectively.

$$\boldsymbol{i} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \boldsymbol{j} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

e.g. $\begin{pmatrix} 4 \\ 3 \end{pmatrix} = 4\begin{pmatrix} 1 \\ 0 \end{pmatrix} + 3\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 4\boldsymbol{i} + 3\boldsymbol{j}$

In general, is $\boldsymbol{a}$ is a vector, then the unit vector $\hat{\boldsymbol{a}}$ in the same direction is

$$\hat{\boldsymbol{a}} = \frac{\boldsymbol{a}}{|\boldsymbol{a}|}$$