

Y13 SPRING 1 - Pure 2 Chapters 2, 3, 4

Chapter 2 - Functions and Graphs

Modulus Functions

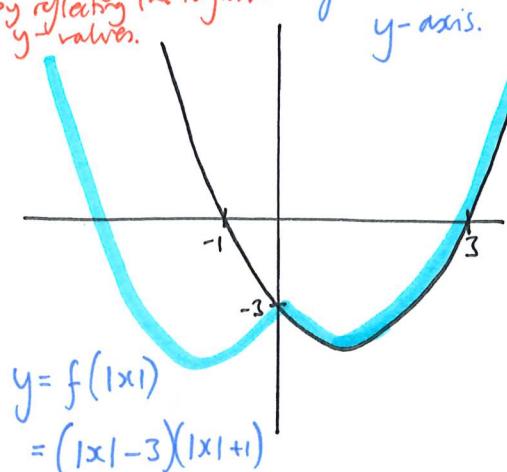
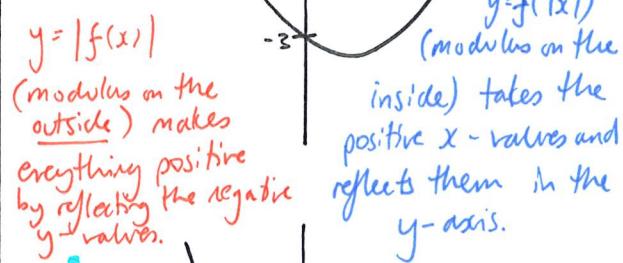
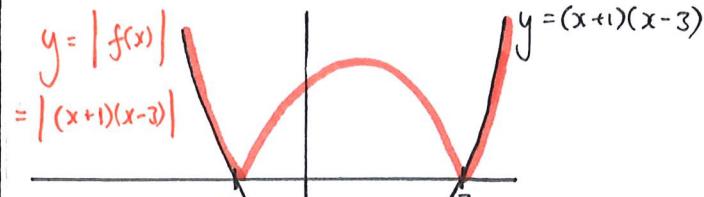
- ① A modulus function is of the type $y = |f(x)|$ modulus (It makes $f(x)$ positive)
 - ② A mapping ~~function~~ is a function if every ~~map~~ input has a distinct input.
- Functions are either one-to-one or many-to-one
- ✓ One-to-one function
 - ✓ Many-to-one function
 - ✗ Not a function
- ③ Composite Functions: $fg(x) = f(g(x))$ means apply g to x first, then apply f to $g(x)$.
- ④ Inverse Functions: $f(x)$ and $f^{-1}(x)$ are inverses of each other.

Therefore $f^{-1}(x)$, and via a reason, the domain of $f(x)$ is the range of $f^{-1}(x)$. The domain and range switch.

As a result of this switch the graphs of inverse functions reflect in the line $y=x$.

SKETCHING FUNCTIONS

Sketching Modulus Functions



Transforming Functions

Inside the bracket	Outside the bracket
$f(x+k)$ $\uparrow \downarrow$ or $\downarrow \uparrow$	$f(x) + k$ $\uparrow \downarrow$ or $\downarrow \uparrow$
Shift	Stretches
Tricky Examples	Note: $f(\frac{1}{3}x)$ means stretch horizontally by a factor of 3 $f(-3x)$ means 'flip horizontally' and then stretch in y by a factor of 3
(Vertical) Reflections	(Vertical) Stretches
Translations	Vertical shifts
Vertical stretches	Vertical reflections
Vertical reflections	Vertical stretches
Vertical reflections	Vertical reflections

fix multiply
column colours

Chapter 3 - Sequences and Series

Arithmetic Sequences

- Difference between terms is constant
- a = first term
- d = common difference
- n = position of a term
- n^{th} term:

$$u_n = a + (n-1)d$$

e.g. $3, 5, 7, 9, 11, \dots$
 $-1, -1.5, -2, -2.5, \dots$

Geometric Sequences

- Ratio between terms is constant
- a = first term
- r = common ratio
- n = position of a term
- n^{th} term:

$$u_n = ar^{n-1}$$

e.g. $2, 6, 18, 54, 162, \dots$
 $\times 3 \quad \times 3 \quad \times 3 \quad \times 3$

$$\text{NB: } \frac{6}{2} = \frac{18}{6} = \frac{54}{18} = \frac{162}{54} = r$$

Series

$$\begin{aligned} \text{If } u_1 = 3 \text{ then } S_1 = 3 \\ u_2 = 9 & \quad S_2 = 3+9=12 \\ u_3 = -1 & \quad S_3 = 12+(-1)=11 \\ u_4 = 2 & \quad S_4 = 11+2=13 \\ & \vdots \end{aligned}$$

So, a series is the "partial sum" of a sequence.

For arithmetic and geometric sequences, the associated series have patterns:

n^{th} term of an ARITHMETIC SERIES:

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

or
 $S_n = \frac{n}{2}(a + l)$ l is the 'last term'
so for S_{17} ,
 $l = u_{17}$

n^{th} term of a GEOMETRIC SERIES:

$$S_n = \frac{a(l-r^n)}{1-r} = \frac{a(r^n-1)}{r-1} \quad (r \neq 1)$$

$$S_\infty = \frac{a}{1-r}$$

Convergent and Divergent Series

$1 + 2 + 4 + 8 + 16 + \dots$ is DIVERGENT

The sum does not tend towards a fixed value

$1 - 2 + 3 - 4 + 5 + \dots$ is DIVERGENT

$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$ is CONVERGENT

← The sum tends towards a fixed value
(in this case 2)

* Arithmetic Series never converge.

* Geometric Series converge if and only if $|r| < 1$

Recurrence Relations

* A recurrence relation of the form $u_{n+1} = f(u_n)$ defines each term of a sequence as a function of the previous term(s) in that sequence.

* With recurrence relation questions, the sequence will likely not be arithmetic or geometric, so previous formulae will not apply.

In this case

A sequence is increasing if $u_{n+1} > u_n$ for all $n \in \mathbb{N}$

A sequence is decreasing if $u_{n+1} < u_n$ for all $n \in \mathbb{N}$

A sequence is periodic if the terms repeat in a cycle

$$(1+x)^n$$

* Binomial expansions, when the index n is negative or fractional, are infinitely long.

* There are 3 common exam questions: ① Combining expansions, ② Accuracy of an expansion ③ Partial Fractions

* To deal with $(4+x)^{\frac{1}{2}}$ we must factorise out the 4 as our form must be $(1+kx)^n$

* The expansion $(a+bx)^n$, where n is fractional or negative,

is valid (convergent) only for $\left|\frac{b}{a}x\right| < 1$ or $|x| < \left|\frac{a}{b}\right|$

Chapter 4 - Binomial Expansions

$$\begin{aligned} \text{e.g. } (4+x)^{\frac{1}{2}} &= \left(4(1+\frac{1}{4}x)\right)^{\frac{1}{2}} \\ \text{now expand this in the usual way.} &= 4^{\frac{1}{2}} \left(1 + \frac{1}{4}x\right)^{\frac{1}{2}} \\ &= 2 \left(1 + \frac{1}{4}x\right)^{\frac{1}{2}} \end{aligned}$$